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Recent Studies on the Response of Structures Subjected to Large Impact Loads

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This paper is dedicated to the memory of Professor John Harvey Evans, who graduated in naval architecture from the University of Liverpool in 1937 and who taught me much about naval architectural structures, when we were colleagues in the Department of Ocean Engineering at the Massachusetts Institute of Technology during the years 1968 - 1979.

Abstract

This article examines some recent studies into the dynamic plastic behavior of structures which are relevant to a wide range of impact and blast loading problems in naval architecture and ocean engineering. Particular emphasis is given to the rigid-plastic methods of analysis which allow surprisingly accurate estimates to be made for the response of structures subjected to blast loads from explosions and to impacts from dropped objects, fragments from burst rotating machinery systems and loose objects propelled by explosive gases. In particular, the accuracy of quasi-static methods of analysis is explored. Quasi-static methods are found to be suitable for simplifying a wide range of structural impact problems in naval architecture and ocean engineering. This article also examines some recent studies into the failure of structures subjected to dynamic loads which cause rupture of the material. Various other topics of interest for safety calculations, hazard assessments and collision resistance in naval architecture and ocean engineering are discussed.

1 Introduction

The modern methods of analysis for the static plastic behavior of structures were introduced into the naval architecture fraternity by Drucker [1] in 1957 and a summary of the progress made up to 1976 was presented in Reference [2] for both static and dynamic loadings. Over the past two decades, a significant expansion has occurred in the applications of plastic methods of analysis to a wide range of static and dynamic structural problems [2-6], including many in naval architecture and ocean engineer-

ing [7]. It is recognized widely that these methods of analysis can provide valuable insight into complex practical problems and may often predict adequate estimates of the principal parameters suitable for design purposes.

The methods of plastic analysis are expected to be valid for predicting the behavior of structures which are subjected to static or dynamic loads producing large plastic strains. They provide estimates of the permanent deformations of structures made from ductile materials and predict the maximum loads which cause failure through tensile tearing or shear sliding due to excessive transverse shear forces in the material. Therefore, an estimate may be made of the maximum possible impact energy absorbed by a structure before the integrity is breached. Moreover, the rigid-plastic procedures are valuable for safety calculations and hazard assessments to select safe operating requirements (e.g., maximum allowable speed of an LNG tanker in a harbor), or in designing adequate protection for critical components to withstand extreme impact events (e.g., main shut-down valve on an offshore platform).

This article focuses largely on recent studies which have been reported on the response of structures subjected to impact loads producing large plastic strains with special emphasis on those structural members used in naval architecture and ocean engineering. Calculations in this particular area provide the response of structures struck by dropped objects, or struck by fragments of burst pressure vessels, pipelines or rotating machinery, or by loose objects which are picked up or which become detached by the rapidly moving gases following an explosion. Other structural problems include the ice and grounding damage of ships and the collision damage and protection of ships and offshore platforms. The same general methods of analysis may be used for the dynamic pressure loading of structures such as the slamming damage of ships and high speed vessels and the blast protection of critical sections and components on offshore platforms.

Reference [8] examines the mass impact loading of structures and discusses some recent studies on a quasi-static method of analysis to simplify this class of problems. The other extreme of explosive loading and the idealization of an impulsive velocity loading for beams and plates is discussed in some detail. The pseudo-shakedown behavior of structures is also examined. This phenomenon may occur when rigid-plastic structures suffer finite-displacements, or geometry changes, when subjected to repeated dynamic loads having identical pressure-time histories. The discussion section in Reference [8] introduces briefly some of the topics of current concern in this field and which are relevant to the structural impact problems in naval architecture and ocean engineering.

It is the objective of the present paper to complement Reference [8] by focusing primarily on some recently published studies. The validity of rigid-plastic methods of analysis is discussed briefly in the next section, while the following four sections on mass impacts explore the range of validity and accuracy of quasi-static methods of analysis, particularly for beams, grillages and pipelines all subjected to large lateral impact loads. The blast and impulsive loading cases are examined in Reference [8], but some discussion is given here in section 7 on the advantages of Youngdahl's correlation parameters when the actual dynamic loading characteristics are unknown, or complicated. Section 8 examines the response of structures due to repeated dynamic loads; sections 9 and 10 focus on the integrity of structures and on the recent development of failure criteria, while section 11 discusses several important aspects which are relevant to the impact behavior of structures in naval architecture and ocean engineering.

2 Validity of Rigid Plastic Methods

A wide range of structural members subjected to dynamic loads had been studied using rigid-plastic methods of analysis which are simplified considerably by disregarding the influence of material elasticity [5, 9]. This is observed to be an acceptable approximation provided large plastic strains are produced giving an energy ratio

$$E_r \gg 1, \quad (1)$$

where E_r is the ratio of the external dynamic energy to the maximum strain energy which may be absorbed by the structure in a wholly elastic manner. Further discussion is given in References [5] and [10] on the minimum acceptable energy ratio, but reasonable predictions have been obtained for values of E_r as small as three when defined in a conservative manner (i.e., the denominator of equation (1) is obtained by assuming that the entire structure yields simultaneously) provided that the pulse duration is smaller than the fundamental period of the structure.

More recently, Symonds and Frye [11] have examined the dynamic behavior of a simple single degree of freedom mass spring model with springs made from either elastic, perfectly plastic, or rigid, perfectly plastic materials. The authors investigated the response due to six different pulse shapes in order to assess the importance of the pulse rise time and pulse duration on the accuracy of the rigid-plastic methods of analysis which disregard all elastic effects. Their conclusions confirm the earlier observation that a large energy ratio is a necessary, but not sufficient condition, for a rigid plastic method to give a good estimate of an elastic plastic solution. Symonds and Frye [11] find that the differences between the elastic, perfectly plastic, and rigid, perfectly plastic analyses may be quite large for pressure pulses having a non-zero rise time and a duration which is long relative to the corresponding natural elastic period. However, the errors are smallest for the largest energy ratios, and the peaks in the error curves decrease with increase in the pulse duration. Furthermore, the errors are small for all dynamic loadings with $E_r > 10$, approximately, and having pulse durations (τ) which are very short compared with the natural period of the system (T), i.e.,

$$\tau/T \ll 1. \quad (2)$$

Yu [12] has examined recently the influence of elastic effects on the dynamic plastic response of a cantilever beam. Yu observed that the inequality (1) is not a sufficient requirement for minimizing the difference between rigid-plastic and elastic-plastic predictions. It transpires that the ratio of the colliding mass to the beam mass plays an important role for impact loads, while Yu [12] again confirms the importance of the pulse shape, rise time and the pulse duration for pulse loaded structures.

An interested reader is directed to Reference [6] where some numerical finite-element studies which assess the importance of material elasticity are discussed briefly.

3 Mass Impact Loading of Beams

This section focuses on the dynamic behavior of beams which are subjected to large impact loads producing material plastic flow. Large impact loads may be caused by accidentally dropping heavy objects into the cargo holds of ships and onto the decks of offshore platforms, for example. In these cases, the impact velocity is

$$V_o = \sqrt{2 g h}, \quad (3)$$

where h is the drop height and g is the acceleration of gravity, while the associated initial impact energy is

$$E_i = G g h = G V_o^2/2 \quad (4)$$

when G is the striker mass. This topic is also relevant for ship and offshore platform collision studies and for assess-

ing the response of offshore structures and protection systems struck by loose objects picked up by the rapidly expanding gases after an offshore explosion or struck by fragments produced during the failure of high speed rotating machinery. In these circumstances, the initial impact velocity is not given by equation (3) and must be obtained by using other means.

The rigid-plastic method of analysis has been used to examine structural impact problems, but many of the theoretical solutions have been simplified by assuming that the displacements remain infinitesimal which is not realistic for the class of practical naval architecture and ocean engineering problems of interest in this article. Finite transverse displacements of initially straight beams and initially flat plates cause geometry changes and introduce membrane forces which exercise an important effect on the dynamic response. However, rigid-plastic methods of analysis have been developed to cater to this phenomenon and good agreement has been obtained with experimental results [5].

The mass impact loading of a fully clamped beam has been examined by several authors as discussed in Reference [8] and the maximum permanent transverse displacement (W_m) of a rigid, perfectly plastic beam with a rectangular cross-section having a fully plastic bending moment M_o and a thickness H is

$$W_m/H = \left\{ (1 + 2 \Omega / 1 + r)^{1/2} - 1 \right\} / 2. \quad (5)$$

according to the studies in References (13) and (14), where $\Omega = G V_o l_1 / 2 M_o H$ and $r = l_1 / l_2$. The span of the beam is $2L = l_1 + l_2$ and the mass G strikes the beam with a transverse velocity V_o at a distance l_1 , from one of the supports. Equation (5) is valid provided $m l_1 / r^2 G \ll 1$, where m is the mass of the beam per unit length.

For the particular case of an impact at the mid-span, equation (5) becomes

$$W_m/H = \left\{ (1 + G V_o^2 L / 2 M_o H)^{1/2} - 1 \right\} / 2. \quad (6)$$

The yield condition used for equations (5) and (6) circumscribes the exact yield curve. Equation (6) and equation (6) for an inscribing yield condition bound almost all of the experimental results from Reference [13] which were obtained on aluminum alloy beams struck at the mid-span, as shown in Reference [8]. Similar agreement between the theoretical predictions and experimental results for non-central impact on aluminum alloy beams is reported in Reference [13]. It is also shown in Reference [8] that the simple procedure proposed in Reference [13] to account for material strain rate sensitivity allows the rigid-plastic method to predict good agreement with the

experimental test results recorded on beams made from mild steel, which is a highly strain rate sensitive material.

4 Quasi-Static Methods of Analysis

The inertia forces are small in many naval architecture and ocean engineering structural problems of interest in this article. These practical problems may be idealized as quasi-static which is a considerable simplification as shown, for example, in section 3.8.4 of Reference [5]. The transverse displacement profile for dynamic loads remains identical to the corresponding one for static loads so that no traveling plastic hinges or time-dependent plastic zones are generated. In fact, this simplification has been used for most ship collision studies [15-18, etc.]. Thus for the mass impact loading case in section 3, a quasi-static estimate for the transverse displacement, W_q , is obtained by equating the initial kinetic energy, E_i , from equation (4), to the work done by a static concentrated load, $P(W)$, at the impact location where the transverse displacement W_q , i.e.,

$$E_i = \int_0^{W_q} P(W) dW \quad (7)$$

Equation (7) is valid for any structural shape whether undergoing finite-displacements, or infinitesimal displacements for which $P(W) = P$.

Naval architects require a criterion to decide whether or not a dynamic structural problem may be examined using quasi-static methods of analysis. This issue was discussed briefly in Reference [19] in the context of ship collisions and it was suggested that a quasi-static method of analysis could be used to examine the dynamic response of ship plating when the duration of the impact forces exceeds the natural period of vibration for the plating. More recently, the accuracy of quasi static methods of analysis has been examined in References [20] and [21].

A simple illustration of the quasi-static method of analysis is given in Reference [21] for a fully clamped beam struck by a mass at the mid-span, as shown in Figure 1. Parkes [22] used a rigid, perfectly plastic analysis to study the dynamic response of the fully clamped beam in Figure 1 which is struck transversely at the mid-span by a rigid mass G traveling with an initial velocity V_o . The theoretical analysis assumes that the plastic yielding of the material is controlled by the magnitude of the bending moment.

It transpires for the exact theoretical solution that the response consists of two phases of motion. For the first phase of motion, two plastic hinges develop at the impact point and travel outwards towards the respective supports where they arrive simultaneously for the particular case of an impact at the mid-span which is examined here. This completes the first phase of motion which is then followed by a second phase of motion with stationary plastic hinges

at the mid-span and at both supports. The permanent deformed profile is reached at the end of the second phase of motion when the initial kinetic energy of the striking mass has been absorbed by the traveling and stationary plastic hinges during the first phase of motion and by the stationary plastic hinges in the final phase of motion.

The maximum permanent transverse displacement when the mass strikes at the mid-span, as shown in Figure 1, is [22]

$$W_m/H = \Omega \left\{ \bar{\alpha}/(1 + \bar{\alpha}) + 2 \log_e (1 + \bar{\alpha}) \right\} / 12\bar{\alpha} \quad (8)$$

where $\bar{\alpha} = mL/G$ and m is the mass per unit length of a beam with a span $2L$ and a plastic bending moment capacity M_0 for the cross-section.

It is evident from Figure 2 that the maximum permanent transverse displacements acquired during the first phase of motion (W_1) are much smaller than those accumulated throughout the second phase (W_2) for mass ratios $G/2mL$ larger than about 5 to 10. This observation occurs because the duration of the first phase of motion for large mass ratios is very short compared with the duration of the second phase of motion.

The external energy which is imparted to the beam in Figure 1 during the first and second phases of motion ($E_e = \int PdW$) is shown in Figures 3(a) and 3(b), respectively. A significant amount of energy is imparted to a beam during the first phase of motion when the mass ratio is small. However, most of the external energy is imparted during the second phase of motion for large mass ratios.

The theoretical predictions in Reference [21] and in Figures 2 and 3 for the impact problem in Figure 1 show that the second phase of motion with stationary plastic hinges at the mid-span and both supports dominates the response for mass ratios ($G/2mL$) larger than about 10 (i.e. $\bar{\alpha} > 0.05$). For example, the permanent transverse displacement acquired during the second phase of motion is about 97% of the total in Figure 2 when $G/2mL = 10$, while the corresponding external energy imparted to the beam is about 91% of the initial kinetic energy (E_i) from Figure 3(b). These observations suggest that the rigid-plastic analysis of many practical engineering problems may be simplified by disregarding the transient or traveling plastic hinge phase of motion.

Now, the static plastic collapse load for a fully clamped beam subjected to concentrated force at the midspan is [5]

$$P_c = 4 M_0/L \quad (9)$$

The external work done by P_c for a transverse displacement W_q underneath the concentrated load is $P_c W_q$, which, for a quasi-static method of analysis, must equal

the initial kinetic energy $E_i = G V_0^2/2$ of the mass in Figure 1, as demanded by equation (7). Thus,

$$w_q = G V_0^2 L/8 M_0 \quad (10)$$

which is independent of the mass of a beam. Equation (8) reduces to equation (10) for large mass ratios (i.e., $\bar{\alpha} \ll 1$). The quasi-static theoretical prediction of equation (10) may also be obtained by equating the initial kinetic energy (E_i) to the energy absorbed by stationary plastic hinges which form at the mid-span and supports as in the second phase of motion of the theoretical analysis [22] leading to equation (8). In this circumstance, the traveling plastic hinge phase of motion in the Parkes solution [22] makes a negligible contribution to the theoretical predictions for W_m as shown in Figure 2 for large mass ratios.

A comparison is presented in Figure 4 between the theoretical predictions from a complete dynamic analysis (equation (8)) and a quasi-static method (equation (10)). It is evident that the quasi-static procedure overpredicts the maximum permanent transverse displacements of a dynamic analysis by 6.6% when $G/2mL = 5$ and that this difference increases to only 10.8 % for a striker which weighs three times the beam mass ($G/2mL = 3$). The quasi-static method of analysis overpredicts the maximum permanent transverse displacement because all of the initial kinetic energy is absorbed in three stationary plastic hinges (i.e., $W_1 = 0, W_2 = W_m$), whereas, in the complete dynamic analysis, the traveling plastic hinges cause plastic energy to be absorbed throughout the entire span as well as at the stationary plastic hinges (i.e., $W_1 \neq 0, W_1 + W_2 = W_m$). This difference in the energy absorbing mechanisms is particularly significant when the striker masses are smaller than the total beam mass. In this case, the traveling plastic hinge phase of motion in the theoretical solution leading to equation (8) is important, as shown in Figure 2. In fact, for $G/2mL = 0.05$ almost all of the initial kinetic energy is absorbed during the first phase of motion with very little remaining to be absorbed during the second or modal phase. It may be shown that the theoretical analysis (22) leading to equation (8) predicts that $W_1 \rightarrow W_m$ and $W_2 \rightarrow 0$ when $G/2mL \rightarrow 0$. Thus, it is clear that quasi-static methods of analysis would be wholly inappropriate when $g/smL < 1$, approximately. A quasi-static method of analysis therefore requires that the mass ratio

$$G/2mL > 1 \quad (11)$$

The theoretical predictions in Figures 2 to 4 were obtained for fully clamped beams subjected to impact loads which produced only infinitesimal displacements. However, it is well known that the influence of finite transverse displacements, or geometry changes, exercise a significant role during the response of fully clamped beams struck by

masses which produce maximum permanent transverse displacements greater than about one-half of the beam thickness [5]. The plastic yielding of a beam undergoing small transverse displacements is controlled by the bending moment with the transverse shear force customarily taken as a reaction force which does not contribute to plastic flow. As the transverse deflections increase, the influence of the bending moment and the transverse shear force diminishes, while the axial membrane force increases until the response approaches that of a plastic string for which plastic flow is controlled by the membrane force alone.

A theoretical study is reported in Reference [20] which compares the predictions of a quasi-static method of analysis according to equation (7) with a dynamic plastic solution for the beam impact problem in Figure 1. This work is discussed further in Reference [8] where it is observed that the influence of finite-deflections, or geometry changes, expands the range of validity of a quasi-static method of analysis. This is particularly noticeable for the larger mass ratios. Thus, an error of less than one per cent is associated with a quasi-static method of analysis for infinitesimal displacements and mass ratios larger than about 33 which reduces to 16.56 for large dimensionless impact velocities when the influence of finite deflections are considered.

5 Mass Impact Loading of Grillages

Theoretical and experimental studies were reported in Reference [23] on the drop weight loading of aluminum alloy and mild steel grillages which are of interest in naval architecture and ocean engineering. The grillages have one main member with two cross-beams, all having rectangular shaped solid cross-sections. The grillages were fully clamped at the supports and struck by a heavy mass at the mid-span with a sufficiently large initial kinetic energy to produce permanent transverse displacements. A rigid, perfectly plastic theoretical analysis for the same grillage subjected to a static concentrated load at the mid-span was developed in Reference (23) although any other available theoretical or numerical procedure would be suitable for the loading on the right hand side of equation (7) for the quasi-static method of analysis.

The permanent transverse displacement profile is obtained for impact loads by equating the work done by the static concentrated load to the initial kinetic energy of the striker, as required by equation (7). It is evident from References [8] and [23] that fair agreement is obtained between the experimental results and the quasi-static theoretical predictions particularly at the junctions between the main and cross beams. The quasi-static predictions at the mid-span were higher than the experimental results which is due, probably, to neglecting the strengthening influence of material strain rate effects.

The initial impact velocities of the strikers in Reference [23] were relatively low and up to 7.34 m/s which may be achieved from a drop height of 2.75 m according to equation (3). A theoretical dynamic analysis for the same rigid, perfectly plastic grillage but when retaining the influence of inertia effect is reported in Reference [24]. This theoretical analysis was used to predict the response for initial impact velocities up to 50 m/s. Figures 5(a)-(c) show the permanent transverse displacement profiles of the main beam for a range of impact velocities and striker masses which maintain a constant initial kinetic energy by reducing the striker mass as the impact velocity increases. The results in Figure 5 and others in Reference [24] indicate that the quasi-static method of analysis is valid for impact velocities which are less than a value lying within the range 10-20 m/s. However it is evident from equation (11) and the results in Reference [21] that the mass ratio should be sufficiently large for the validity of a quasi-static method of analysis although a mass ratio as small as 1.59 in a mass impacted beam leads only to an error of 10 percent in the maximum permanent transverse displacements. The mass ratios are 66.7, 4.2 and 0.67 for the results in Figures 5(a)-(c), respectively.

6 Mass Impact Loading of Pipelines

This case is somewhat similar to the beam impact problem examined in section 3 and section 4 except it is now necessary to cater to the distortion of the pipeline cross-section. Some experimental results for steel pipelines, which are fully clamped across a span and struck laterally by a rigid mass with impact velocities up to 14m/s, are presented in Reference [25] and discussed in Reference [8]. This particular case is of interest for a number of practical drop weight impact problems in naval architecture and ocean engineering and for the collision protection of offshore platforms.

It transpires that a theoretical rigid-plastic analysis [26] using the quasi-static assumption given by equation (7) and discussed in section 4 was found to give good agreement with the corresponding experimental results over the whole range of outside diameter to thickness ratios (D/H), as shown in Figure 6 for impacts at the mid-span. This theoretical method retains the influence of the local or denting deformations throughout the global response phase which has not been done in previous theoretical studies except from an empirical viewpoint in Reference [27].

Several other rigid-plastic methods of analysis have been developed for this problem and predict the results shown in Figure 6. The theoretical procedure in Reference [28] neglects the distortion of a pipeline cross-section and underpredicts the maximum permanent transverse displacements except for $D/H = 11$. Ellinas and Walker [29] use a semi-empirical method of analysis and overpredict

the corresponding experimental values except for $D/H = 30$. The approximate procedure in Reference [30], the details of which have not been published, along with the theoretical analysis in Reference [26], provide the best agreement with the experimental results in Reference [25]. Moreover, Reference [26] contains the only method of analysis which caters for strikes at any location on the span, except very close to a support and it is shown in Reference [26] to predict good agreement with the experimental results which were recorded on steel pipelines at the one-quarter span position.

The threshold impact energy which is required to produce a pipeline failure was also recorded in Reference [25]. Failure may occur in a variety of modes but most would allow the contents to escape producing a potentially hazardous situation. It is particularly noteworthy from these tests that the energy required to produce a failure for impacts close to a support is significantly smaller than that required for failure at the mid-span.

7 Blast Loadings

The previous sections focused on the response of structures struck by large masses which, for naval architecture and ocean engineering problems, generally travel at fairly low velocities, so that the structural response may be idealized as quasi-static and analyzed using equation (7). At the other extreme, blast type loads may originate due to explosions on offshore platforms or on board ships when loading or transporting hazardous cargoes. In this case, the transverse inertia forces in the structural members cannot be neglected and cause time-dependent transverse displacement profiles with traveling plastic hinges and plastic zones.

Theoretical rigid-plastic analyses have been developed for the response of various structures subjected to pressure-time histories with the rectangular shaped pressure-time history being the most common form of loading, as observed in Reference [8]. However, for large dynamic pressure pulses having a short duration, the magnitude of the pulse is important while the actual shape of the pulse does not play a significant role. In fact, the dynamic pressure pulses having a peak pressure larger than about ten times the corresponding static collapse pressure may be idealized as impulsive with little sacrifice in accuracy.

A uniform impulsive velocity loading, V_0 , for a structure having an area A subjected to a uniform pressure of magnitude p_0 with an extremely short duration, τ , is obtained from the requirement to conserve linear momentum [5]

$$\mu A V_0 = p_0 A \tau,$$

or

$$V_0 = p_0 \tau / \mu \quad (12)$$

when $p_0 / p_c \gg 1$ and $\tau \rightarrow 0$ and where μ is the structural mass per unit area and p_c is the associated static plastic collapse pressure. It is simpler to solve an impulsive loading problem because it avoids a phase of motion while the pressure pulse is active, which, therefore, reduces the number of phases of motion in a theoretical analysis. The initial kinetic energy of an impulsive loading is

$$E_i = \mu A V_0^2 / 2. \quad (13)$$

This simplification has been used in many rigid-plastic analyses which have given good agreement with the experimental results obtained on explosively loaded beams and circular and rectangular plates [5].

The impulsive loading of a fully clamped rectangular plate is an important practical naval architecture and ocean engineering structure and has been examined in several References [5, 31]. The maximum permanent transverse displacement is

$$\frac{W_m}{H} = \frac{(3 - \epsilon_0) \{ [1 + \lambda \epsilon_0^2 (1 - \epsilon_0 + 1/(2 - \epsilon_0))/6]^{1/2} - 1 \}}{2 [1 + (\epsilon_0 - 1)(\epsilon_0 - 2)]} \quad (14)$$

where $\lambda = \mu V_0^2 L^2 / M_0 H$, $\epsilon_0 = \beta \{ (3 + \beta^2)^{1/2} - \beta \}$, μ is the mass per unit area of the plate, β is the plate aspect ratio ($0 \leq \beta \leq 1$) and $2L$ is the length of the longer side. Equation (14) with $\beta \rightarrow 0$ predicts

$$W_m / H = \{ (1 + 3\lambda/4)^{1/2} - 1 \} / 2 \quad (15)$$

for a fully clamped beam subjected to a uniformly distributed impulsive velocity [5]. Equations (14) and (15) give good agreement with experimental results which have been recorded on aluminum alloy beams and plates [5]. The important influence of material strain rate sensitivity has also been examined for mild steel beams and plates loaded impulsively, as discussed in Reference [5]. The same method of analysis has been used to examine the simply supported case.

In some practical cases the dynamic pressure loadings cannot be idealized as impulsive. Furthermore, it may be shown for relatively small pressure pulses $1 \leq p_0 / p_c \leq 10$, approximately, that the response of a structure is sensitive to the time-dependence of the pressure pulse [8]. In order to overcome this difficulty, Youngdahl [32] has studied the influence of pulse shape on the dynamic plastic response of rigid-plastic structures. He observed that the maximum permanent transverse displacement depends significantly upon the pulse shape. However, Youngdahl found that the strong dependence on pulse shape could be practically eliminated by introducing an effective load

$$P_e = 1/2t_c, \quad (16)$$

where

$$I = \int_{t_y}^{t_f} P(t)dt \quad (17)$$

is the total impulse, $P(t)$ is the external load, t_y and t_f are the times when plastic deformation starts and finishes and where t_c is the centroid of the pulse given by

$$It_c = \int_{t_y}^{t_f} (t - t_y) P(t)dt. \quad (18)$$

For the purpose of estimating the response time t_p , which is not known a priori, Youngdahl [32] replaced equation (17) by the simpler expression

$$I \approx P_c (t_f - t_y), \quad (19)$$

where P_c is the corresponding static plastic collapse load.

The correlation parameters for the effective load (P_c), the total impulse (I) and the mean time (t_c), which are defined by equations (16) to (18), respectively, contain integrals of the external loading and are, therefore, insensitive to small perturbations in the pulse shape. This accounts for the essential collapse onto a single curve of the theoretical predictions for rigid plastic structures subjected to different pulse shapes. These observations are encouraging from a practical viewpoint because it is often difficult to record accurately the pressure-time histories of dynamic loadings and to model the actual dynamic loads in laboratory tests. Thus, a designer may use the simplest available theoretical analysis for the response of a structure (e.g., a rectangular pressure pulse loading) together with the correlation parameters given by equations (16) - (18) to predict the structural response for the same structure subjected to any complex pressure-time history.

8 Repeated Dynamic Loads

Naval architecture and ocean engineering structures are sometimes subjected to repeated dynamic loadings. For example, this situation occurs during the bow and bottom slamming and ice damage of ships and marine vehicles and the wave action on the supporting structures of offshore platforms.

The effect of repeated static loadings, which are large enough to cause repeated plastic flow in structural members, has been studied extensively by many authors [33]. In certain circumstances, a shakedown state may be achieved when plastic flow ceases and a wholly elastic behavior is associated with any further repeated and identical static loads. This phenomenon is well defined for an elastic, perfectly plastic material and is illustrated in Reference [34] for the repeated hogging and sagging of a ship

hull. A similar phenomenon may occur for repeated dynamic loadings on elastic, perfectly plastic structures.

The studies on the dynamic response of the rigid plastic structural members reported in the earlier sections emphasized the significance of finite transverse displacements and geometry changes and the unimportance of elastic effects. Thus, the phenomenon of pseudo shakedown was introduced in Reference [35] for this class of structures and illustrated for a rigid, perfectly plastic rectangular plate subjected to repeated dynamic pressure pulses. Pseudo-shakedown may develop only in a rigid-plastic structure which is subjected to identical repeated loads producing stable finite deflections (e.g., axially restrained beams, circular, rectangular and arbitrarily shaped plates, and axially restrained cylindrical shells).

A conjecture on the pseudo-shakedown phenomenon for beams and plates is reported in Reference [36] and discussed and illustrated in Reference [8] to which an interested reader is referred for further information.

9 Structural Failure

The previous sections have examined the behavior of structures subjected to impact loads which produce large ductile deformations. Thus, the methods of analysis may be used to predict the impact energy absorbed by a structure and the associated magnitude of the permanent deformations when assuming that the material has an unlimited ductility. No information is obtained, therefore, on the structural integrity. However, this aspect is important for many problems in naval architecture and ocean engineering. For example, it is necessary for designers to estimate the damage associated with the collision and grounding of ships causing breaches in the structural integrity leading to the release of hazardous materials which may be harmful to the environment and to people. Designers also need to assess the integrity of critical components on offshore platforms which could be breached in various accident scenarios involving dynamic events.

A survey on the dynamic inelastic failure of beams was presented in Reference [37]. The beams were made from ductile materials, which could be modelled as rigid, perfectly plastic, and they were subjected to either uniformly distributed impulsive velocity loads, as an idealization of an explosion, or mass impacts to idealized dropped object loading. The total external dynamic energy for an impact load is given by equation (4), while equation (13) for an impulsive velocity loading with $\mu A = m2L$ may be written

$$E_i = mL V_0^2 \quad (20)$$

for a beam of length $2L$ and a mass, m , per unit length. It was assumed that the energy imparted by the external dynamic loads according to equations (4) and (20) satisfy

inequality (1). Thus, large plastic strains were produced and the possibility of material rupture was studied for sufficiently large dynamic loads.

It was observed that the different dynamic loadings examined in Reference [37] may cause the development of different failure modes. The simplest failure modes were associated with a uniformly distributed impulsive velocity loading. A rigid-plastic method of analysis for this particular problem [5] shows that membrane forces as well as bending moments must be retained in the basic equations for the response of axially restrained beams subjected to large dynamic loads which cause transverse displacements exceeding the beam thickness, approximately. This is known as a Mode I response with the associated design requirements

$$E_1 \leq s_1 E^* \quad (21)$$

and/or

$$W_m \leq s_2 W^*, \quad (22)$$

where E^* and W^* are the maximum energy which may be absorbed plastically and the maximum permanent transverse displacement of a beam without material failure, respectively. The parameters s_1 and s_2 in equations (21) and (22) are design safety factors which satisfy the inequalities $0 < s_1 \leq 1$ and $0 < s_2 \leq 1$. Equation (15), for example, was developed for the Mode I response of a beam subjected to an impulsive load.

If the external impulse is severe enough then the large strains which are developed at the supports of an axially restrained beam would cause rupture of the material which is known as a Mode II failure, i.e.,

$$\epsilon_{mx} = \epsilon_r \quad (23)$$

where ϵ_{mx} is the maximum strain developed in the beam and ϵ_r is the uniaxial rupture strain of the material. At still higher impulsive velocities, the influence of transverse shear forces dominates the response and failure is more localized and occurs due to excessive transverse shearing displacements (Mode III). Thus, this failure mode occurs when

$$W_s = kH \quad (24)$$

where W_s is the transverse shear displacement at the support, H is the beam thickness and k is an experimental parameter having a value within the range $0 < k \leq 1$.

It is important to note that a Mode III transverse shear failure is more likely to occur in dynamically loaded beams than in similar statically loaded beams. For example, infinite transverse shear forces are generated on the application of an ideal impulsive loading to a rigid-plastic

beam with plastic yielding controlled by the bending moment alone, while transverse shear forces must remain finite in a similar uniformly loaded static beam problem to equilibrate with the external static load. Thus, even a beam with a solid rectangular cross-section and a large length-to-thickness ratio can suffer a transverse shear failure under a dynamic loading (Mode III response), as observed by Menkes and Opat [38] and analyzed in Reference [39]. This effect is even more pronounced for beams with open cross-sections which are found throughout naval architecture and ocean engineering structures.

Paradoxically, it is observed in Reference [37] that despite the lower impact velocities of the mass impact case, the failure behavior is much more complex than the impulsive loading case. The Mode II and III failure modes discussed above for an impulsive loading also occur for a mass impact loading. However, it transpires that other more complex failure modes may develop. For example, a striker may cause an indentation on the struck surface of a beam, which, if sufficiently severe, may lead to failure. The impact velocity of a strike near to the support of a beam might not be sufficiently severe to cause a transverse shear failure (Mode III) but could distort severely a beam and cause rupture due to the combined effect of transverse shear force, membrane force and bending moment, as shown in Figure 14 of Reference [37].

It is evident [37] that the dynamic inelastic rupture of beams and other structures is an extremely complex phenomenon and that there is a pressing need for the development of a reliable criterion which can be used in theoretical methods, numerical schemes and computer codes in order to predict the onset of structural failure due to material rupture for hazard assessments and safety calculations throughout the field of naval architecture and ocean engineering structures.

In an attempt to obtain a universal failure criterion, which could be used for a large class of dynamic structural problems, an energy density failure criterion was introduced in Reference [40] and discussed in Reference [41]. It is assumed that rupture occurs in a rigid-plastic structure when the absorption of plastic work (per unit volume), θ , reaches the critical value

$$\theta = \theta_c, \quad (25)$$

where θ contains the plastic work contributions related to all of the stress components. The maximum possible value of θ_c is taken as

$$\theta_{cm} = \int_0^{\epsilon_{to}} \sigma_d(\epsilon, \dot{\epsilon}_m) d\epsilon, \quad (26)$$

where $\sigma_d(\epsilon, \dot{\epsilon}_m)$ is the dynamic engineering stress-strain curve which is obtained from a dynamic uniaxial tensile

test for a given $\dot{\epsilon}_m$ and where ϵ_{to} is the engineering rupture strain (zero gauge length). In general, both ϵ_{to} and σ_d could depend on the magnitude of the strain rate [37, 42]. For a rigid, perfectly plastic material, equation (26) simplifies to

$$\theta_{cm} = \sigma_{do} \epsilon_p \quad (27)$$

when

$$d\epsilon_p = \left(\frac{1}{\sigma_{do}} \right) \int_0^{\epsilon_{to}} \sigma_d(\epsilon, \dot{\epsilon}_m) d\epsilon \quad (28)$$

and where σ_{do} is the mean dynamic flow stress.

In the particular case of a rigid-plastic beam having a width B and a thickness H , equation (25) gives the actual plastic work

$$\gamma = \gamma_c = \theta_c B H l_m = \sigma_{do} B H l_m \quad (29)$$

absorbed at failure in a plastic hinge having an average length l_m across the beam thickness, where ϵ_c is a critical strain, the maximum value of which is ϵ_p given by equation (28). Equation (29) may be recast into the dimensionless form

$$\gamma^* = \gamma_c^* = \gamma_c / \sigma_{do} B H l_m = \epsilon_c \quad (30)$$

The Cowper-Symonds constitutive equation may be used to give the mean dynamic flow stress [5]

$$\sigma_{do} = \sigma_o \left\{ 1 + \left(\dot{\epsilon}_m / C \right)^{1/q} \right\} \quad (31)$$

where C and q are material constants, σ_o is the static uniaxial yield stress and $\dot{\epsilon}_m$ is the mean strain rate throughout the response.

The mean hinge length l_m across the beam depth in equations (29) and (30) is taken in Reference [40] as

$$l_m = \alpha H. \quad (32)$$

where α depends on the shear work parameter

$$\Psi = \gamma_s / \gamma, \quad (33)$$

and where $d\gamma_s$ is the plastic work absorbed in a plastic hinge through shearing deformations and γ is the total amount of plastic work absorbed at the same plastic hinge.

The fully clamped impulsively loaded beam, which was studied experimentally by Menkes and Opat [38] and theoretically with a rigid-plastic method of analysis in Reference [39] was re-examined in Reference [40] using the critical density failure criterion which is governed by equations (25) - (33). If $\gamma = \gamma_c$ at a plastic hinge at any time during the response, where γ_c is defined by equation (29)

then a Mode II response occurs, or possibly, a Mode III response. It is proposed in Reference [40] that a Mode III transverse shear failure occurs when $\gamma = \gamma_c$ as for a Mode II failure, but with $\psi = \psi_c$, where ψ is defined by equation (33). The transition between a Mode II and a Mode III failure may also depend on other factors, including the shape of a beam cross-section. The actual value of ψ_c for a Mode III failure must be found from comparisons between the theoretical predictions and the corresponding experimental results.

The results in Reference [40] show that α and ψ in equations (32) and (33) are related by the approximate straight line

$$\alpha + 1.2 \Psi = 1.3. \quad (34)$$

Thus, for $\Psi = 0$ (i.e., no transverse shear deformations, $\alpha = 1.3$, so that the plastic hinge is 30 percent longer than the beam thickness. It is evident from equation (34) that α decreases as Ψ increases which is also anticipated from a physical viewpoint.

The numerical predictions of the energy density failure criterion in Reference [40] for the dimensionless impulses at the transitions between a Mode I and Mode II failure and between a Mode II and a Mode III failure are compared in Table 1 with the corresponding experimental results of Menkes and Opat [38] and the theoretical rigid, perfectly plastic predictions in Reference [39]. Many other theoretical predictions for the various parameters are presented in References [40] and [41] for an impulsively loaded beam to which an interested reader is referred. The energy density failure criterion predicted good agreement with the available experimental results on impulsively loaded beams and confirmed broadly the failure characteristics which were first observed using the elementary analysis developed in Reference [39].

The critical energy density failure criterion represented by equations (25) - (33) was also used in Reference [43] to examine the failure of fully clamped beams subjected to mass impact loads. The rigid-plastic beams were fully clamped across a span of length $2L$ and struck at a distance l_p from the nearest support by a mass G traveling with an initial impact velocity V_o .

Figure 7 compares the theoretical predictions from Reference [43] with the corresponding experimental results reported in Reference [13] for a Mode II tensile tearing failure.

Some recent experimental [44, 45] and theoretical [46] studies have been reported on the dynamic inelastic failure of fully clamped circular plates subjected to uniformly distributed impulsive loads. The influence of the boundary conditions on the magnitude of the critical impulse is

Surface Crack Model for $a/2c = 0.15''$, $0.25''$, or $0.35''$ and $2c = 2\text{-inch}$ or 3-inch

Through Thickness Crack Growth Model.

Crack Grows from \sim to $2a_{cr} = 15''$ at $2a_{cr}$ - Rapid Fracture Occurs.

The Effect of 2C Value on the Crack Growth Time Using the Reduced Stress (0.7 stress RMS)

shown to be very important, which, in fact, has already been observed for beams [13, 47]. This complicates further the predictions of an energy density failure criterion but shows the importance of specifying carefully the exact details of the boundary conditions in both experimental tests and theoretical studies.

The entire area of structural failure due to material rupture is a very important one in naval architecture and ocean engineering, but the present state of knowledge is incomplete from both experimental and theoretical viewpoints. The simple rigid-plastic methods of analysis with the criteria represented by equations (21) to (24) have, in fact, been quite successful for predicting the failure of several structural problems which have been examined and they are particularly useful for preliminary design. The energy density failure criterion is also promising for predicting the failure of a broader range of structural problems but it is more difficult to use. Eventually, it could be incorporated into any theoretical method or numerical scheme, but in sufficient information is available currently to do this with confidence except for beams. It is often assumed that rupture occurs when the equivalent strain in a structural member reaches the rupture strain recorded in a uniaxial tensile test. It is important to emphasize that, generally speaking, this assumption is incorrect [48].

Holmes et al. [49] have used a local damage model in a DYNA 3D computer program in order to predict the ductile fracture of a welded steel T-joint subjected to dynamic loading. This method appears promising and could be developed for further naval architecture and ocean engineering structural problems. The fracture initiation and the fracture propagation path through a structure are obtained using the normalized damage parameter

$$\Delta = \int \frac{d \epsilon_e p}{\epsilon_c (\sigma_m / \sigma_e)} = 1, \quad (35)$$

where $d \epsilon_e p$ is a plastic strain increment and $\epsilon_c (\sigma_m / \sigma_e)$ is the critical strain as a function of the stress triaxiality which is defined as the ratio of the mean stress (σ_m) to the equivalent stress (σ_e). This criterion assumes that failure occurs when the integral of the equivalent plastic strain increments in a characteristic volume of the material equals a critical failure strain which is a function of the stress triaxiality.

10 Perforation of Structures

The perforation of structures has been studied extensively for many years but largely for high velocity missiles in military and aerospace engineering applications [50 - 52]. However, accident scenarios involving perforation may also occur in naval architecture and ocean engineering. For example, the rapidly moving gases which follow an explosion on an offshore platform may break away various fittings and pick up loose objects and propel them onto critical components leading to perforation. It is likely that the velocity of the smaller mass objects may fall within the range of validity of the available well-known empirical equations which have been generated for perforation. However, the impact velocities of the larger objects might lie outside the range of validity which, therefore, must be examined and, if necessary, new equations developed.

Various objects may be dropped onto the structural members in ship and offshore platforms and cause perforation of critical components. The impact velocities of these projectiles or missiles are given by equation (3) and would often fall outside the range of validity of the various empirical formulae which have been developed to predict perforation. Recently, several research groups have con-

ducted studies into the perforation of plates in order to provide design formulae for civilian applications at lower impact velocities.

Reference [53] has examined the perforation of mild steel plates struck at the mid-span by blunt or flattened projectiles at low speeds and developed the new empirical formula for the perforation energy (Nm)

$$E_p/\sigma_u d^3 = (2\sigma_y/\sigma_u) [(\pi/4)(H/d)^2 + (S/d)^{0.21}(H/d)^{1.47}] \quad (36)$$

where σ_y and σ_u are the yield and ultimate tensile stresses (N/m²) of the mild steel target plate and d, S and H are the projectile diameter (m), unsupported span (m) and plate thickness (m), respectively. Equation (36) gives good agreement with the experimental results reported by Langseth and Larsen [54], Corran et al. [55], Neilson [56] as well as the new experimental data reported in Reference [53].

11 Discussion

Several important topics, which are relevant to various safety studies and hazard assessments involving impact loads in naval architecture and ocean engineering, are discussed in the previous sections. These topics form part of a much larger research field into the dynamic plastic behavior of structures, energy absorbing systems and structural crashworthiness. It would be difficult to survey this entire field, but current progress in some areas of relevance to naval architecture and ocean engineering are introduced briefly in this section.

The rigid-plastic methods of analysis, which are emphasized in this article, are valuable for preliminary design calculations and are often adequate for final designs because of uncertainties in the dynamic material properties, external loading characteristics and the structural details. However, numerical schemes or standard computer codes are sometimes necessary for complex problems. Moreover, in most critical or sensitive cases either full-scale prototype or small-scale model experimental tests are necessary in order to calibrate both the theoretical methods and the numerical codes because of the paucity of accurate data and the highly nonlinear nature of the problems which are on the forefront of research into the response and failure of materials and structures under dynamic loads.

Considerable experimental and theoretical effort has been expended over the last several decades into studying the strain rate sensitive behavior of materials. Most of the tests have been conducted on materials subjected to uniaxial stress states producing relatively small strains only up to several per cent because of the experimental difficulties which are encountered when conducting con-

trollable and repeatable tests with large strains at high strain rates. However, most of the important practical applications for naval architecture and ocean engineering structures are concerned generally with dynamic loads which produce large plastic strains often up to rupture, as in ship collision and grounding studies, for example. This topic is discussed further in Reference [42] and the constitutive equation

$$\frac{\sigma_{do}}{\sigma_o} = 1 + \left\{ \frac{\dot{\epsilon}}{B + F\epsilon} \right\}^{1/q}, \quad \epsilon_y \leq \epsilon \leq \epsilon_u, \quad (37)$$

where

$$B = C - F\epsilon_y \quad (38)$$

and

$$F = \frac{C_u - C}{\epsilon_u - \epsilon_y} \quad (39)$$

is proposed in order to cater to the variation of the strain rate effect on the flow stress with increasing strain. σ_{do} and σ_o are the respective dynamic and static flow stresses, $\dot{\epsilon}$ is the uniaxial strain rate and ϵ_y and ϵ_u are the yield and ultimate strains, respectively. Equation (37) with $\epsilon = \epsilon_y$ reduces to the familiar form [5] of the Cowper-Symonds relation when C and q are identified with the usual coefficients for small strains. Equation (37) with $\epsilon = \epsilon_u$ for large strains again takes on the usual form of the Cowper-Symonds equation except that the coefficient C_u is evaluated from the strain rate sensitive properties at the ultimate tensile strength of the material.

The rupture strains of some materials also change with strain rate and for mild and stainless steels it was shown that the dynamic rupture strains are [37, 42]

$$\epsilon_{dr} = \left\{ 1 + (\dot{\epsilon}/C)^{1/q} \right\}^{-1} \epsilon_r, \quad (40)$$

where ϵ_r is the static uniaxial rupture strain.

The values of the various constants in equations (37)-(40) are given in Reference [42] for mild steel and for equation (40) in Reference [57] for stainless steel. Further experimental work is required in order to provide more reliable data for the various material constants in equations (37) and (40) and to ensure that the forms of these equations are adequate for design purposes.

Other studies have been reported into many aspects of the general area such as dynamic plastic buckling, energy absorbing systems, impact perforation, transverse shear and rotatory inertia effects and investigations into the scaling laws for the impact loading of structures.

Conclusions

It is the aim of this article to present the current status of studies on the impact loading of structures which are relevant for the analysis of various dynamic structural problems in naval architecture and ocean engineering. In particular, the accuracy of quasi-static methods of analysis are examined and it is observed that they can be used to simplify many practical problems without sacrifice in accuracy provided the impact mass is much larger than the mass of the struck structural member. Youngdahl's correlation parameters were also found to be valuable for simplifying theoretical analyses by eliminating the requirement for an accurate representation of a blast loading. Several equations are presented which can be used for preliminary design purposes and yet are sufficiently accurate for many final designs because of the paucity of data on the dynamic material properties and on the dynamic load characteristics.

Current knowledge in the field of dynamic structural plasticity may be used to provide insight into and solutions for many naval architecture and ocean engineering impact problems. However, further research is still required in several areas including the dynamic properties of materials at large strains.

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Notation

d	diameter of a projectile	H	beam, grillage, plate or pipe wall thickness
g	gravitational constant	I	total impulse
h	drop height	I*	$L/\{A(\rho\sigma_y H^2)^{1/2}\}$
k	defined by equation (24)	2L	span of a beam or pipeline, or length of a rectangular plate
l_1	distance of impact point from the nearest support	M_0	fully plastic bending per unit length of a beam or a plate cross-section
l_2	$2L-l_1$	P	concentrated force
l_m	average plastic hinge length across the thickness of a beam	P_c	concentrated static plastic collapse force
m	mass per unit length of a beam	P_e	effective load
p_0	magnitude of a pressure pulse	S	span of a plate
p_c	static plastic collapse pressure	T	natural period of vibration
q	Cowper Symonds exponent in equation (31)	V_0	impact velocity of a mass or impulsive velocity
r	l_1 / l_2	W	transverse displacement
s_1, s_2	safety factors	W_1, W_2	maximum permanent transverse displacements during the first and second phases of motion, respectively.
t	time	W_m	maximum permanent transverse displacement
t_c	defined in equation (18)	W_q	maximum permanent transverse displacement predicted by a quasi-static method of analysis
t_f	duration of response	W_s	transverse shear displacement
t_y	time when plastic yielding commences	W^*	maximum permanent transverse displacement without material failure
A	surface area of a structural member	α	non-dimensional hinge length defined by equation (32)
B	width of a beam	$\bar{\alpha}$	mL/G
C, C_u	Cowper Symonds coefficient in equation (31) for yield and ultimate stresses, respectively.	β	aspect ratio of a plate ($0 \leq \beta \leq 1$)
D	outside diameter of a pipeline	γ	plastic work absorbed in a plastic hinge in a rigid-plastic beam
E_e	total external energy	γ_c	critical value of γ
E_i	initial energy	γ^*, γ_c^*	non-dimensional values of γ and γ_c as defined by equation (30)
E_p	perforation energy		
E_r	energy ratio		
E^*	maximum energy which may be absorbed plastically without material failure		
G	mass of a striker		

γ_s	plastic work absorbed in a plastic hinge through shearing deformations	μ	mass per unit area
ε	strain	ξ	dimensionless position of a travelling plastic hinge
$\dot{\varepsilon}$	strain rate	ξ_0	$\beta \{ (3 + \beta^2)^{1/2} - \beta \}$
ε_c	critical strain	ρ	density of material
ε_{dr}	dynamic rupture strain	σ_d	dynamic flow stress
ε_e	equivalent strain	σ_{do}	mean dynamic flow stress
ε_{fo}	engineering rupture strain (zero gauge length)	σ_e	equivalent stress
$\dot{\varepsilon}_m$	mean strain rate	σ_m	mean stress
ε_{mx}	maximum strain	σ_o	static flow stress
ε_p	defined by equation (28)	σ_u	static ultimate stress
ε_r	static rupture strain	σ_y	static yield stress
ε_u	ultimate strain	τ	pulse duration
ε_y	yield strain	Ψ	γ_s/γ
θ	plastic energy absorbed per unit volume of material	Ψ_c	critical value of Ψ
θ_c	critical value of θ	Δ	normalised damage parameter given by equation (35)
θ_{cm}	defined by equation (26)	Ω	$G V_0^2 l_1 / 2 M_0 H$
λ	$\mu V_0^2 L^2 / M_0 H$		

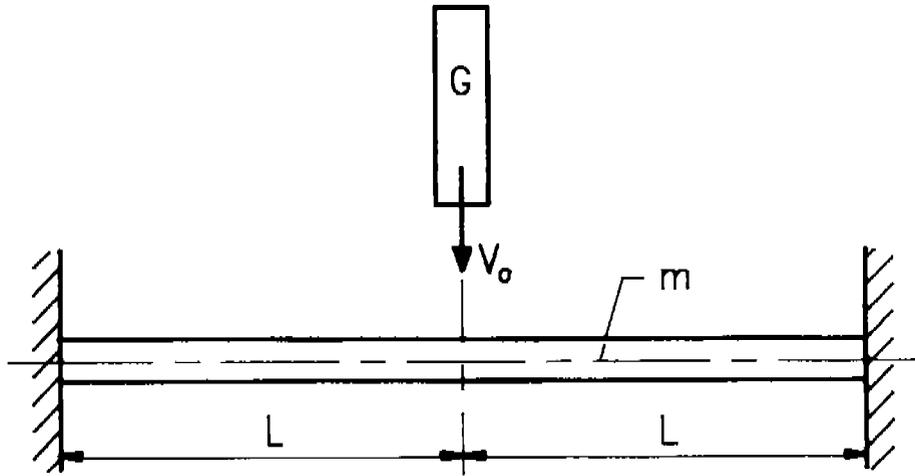


Figure 1

Fully clamped rigid, perfectly plastic beam having a span $2L$ and a mass m per unit length and struck at the mid-span by a mass G travelling with an initial velocity V_0 .

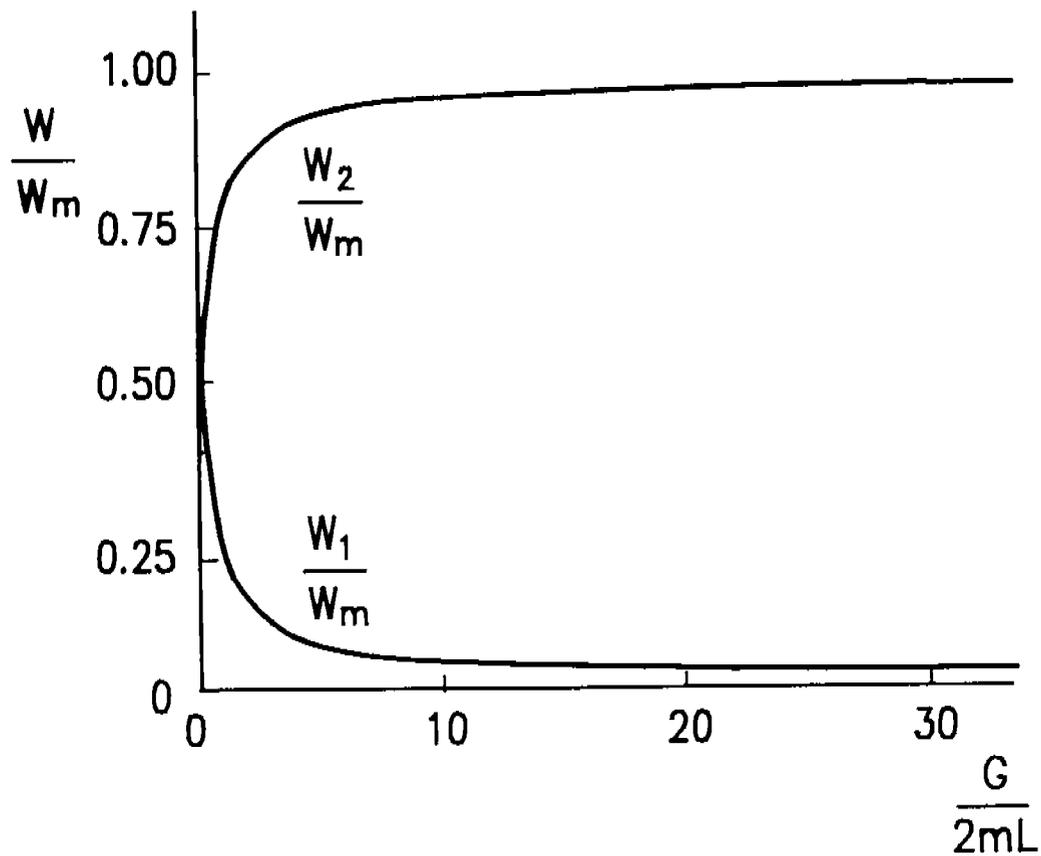


Figure 2

Dimensionless permanent transverse displacements at the end of the first phase of motion (W_1) and acquired during the second phase of motion (W_2) for the beam in Figure 1. W_m is the total permanent transverse displacement given by equation (8).

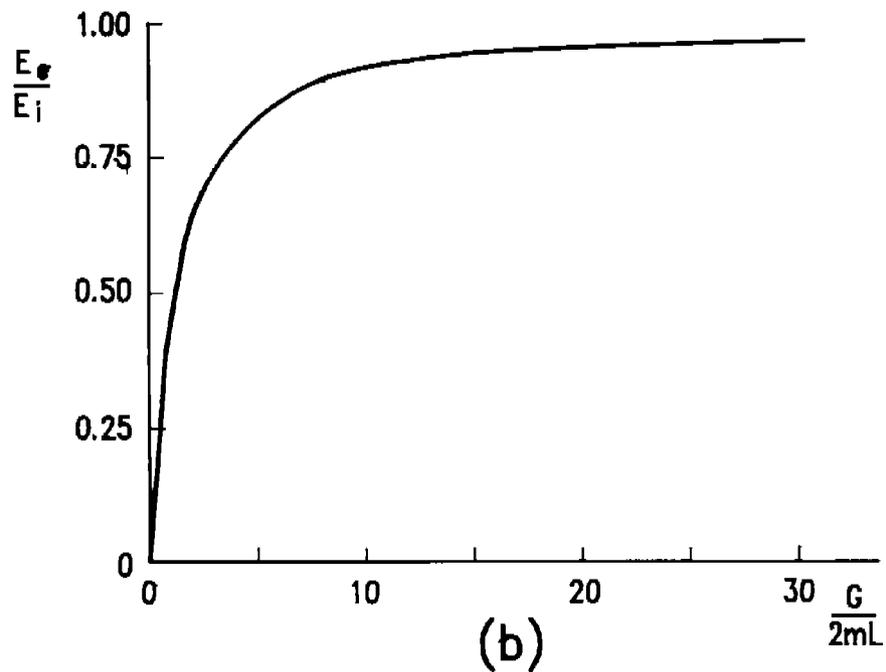
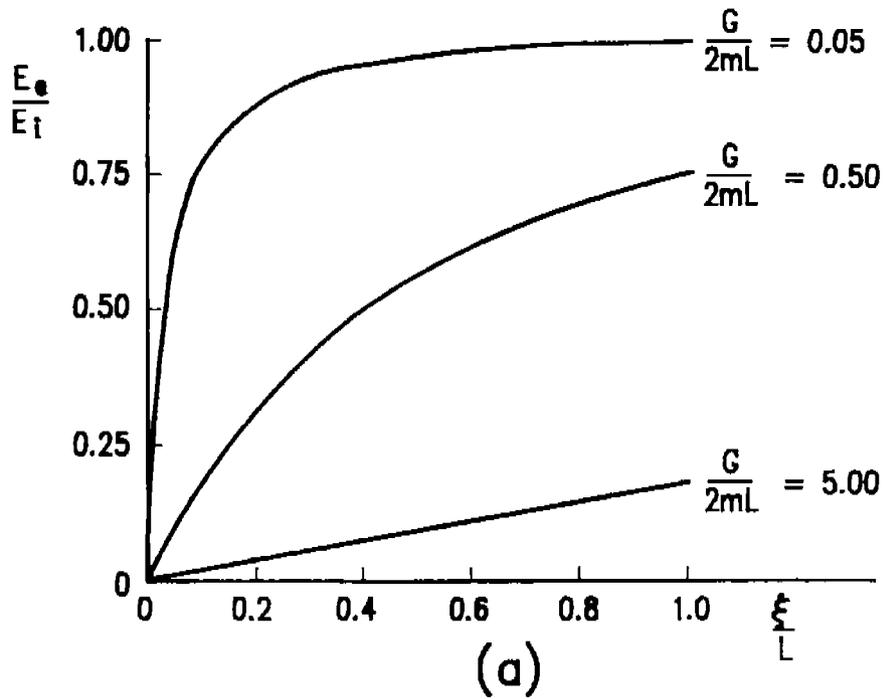


Figure 3

(a)

Dimensionless external energy (E_e) imparted to the beam in Figure 1 as a function of the dimensionless position (ξ , $0 \leq \xi \leq L$) of the travelling plastic hinges during the first phase of motion. $E_i = G V_0^2/2$ is the initial kinetic energy of the mass G .

(b)

Dimensionless external energy (E_e) imparted to the beam in Figure 1 during the second phase of motion.

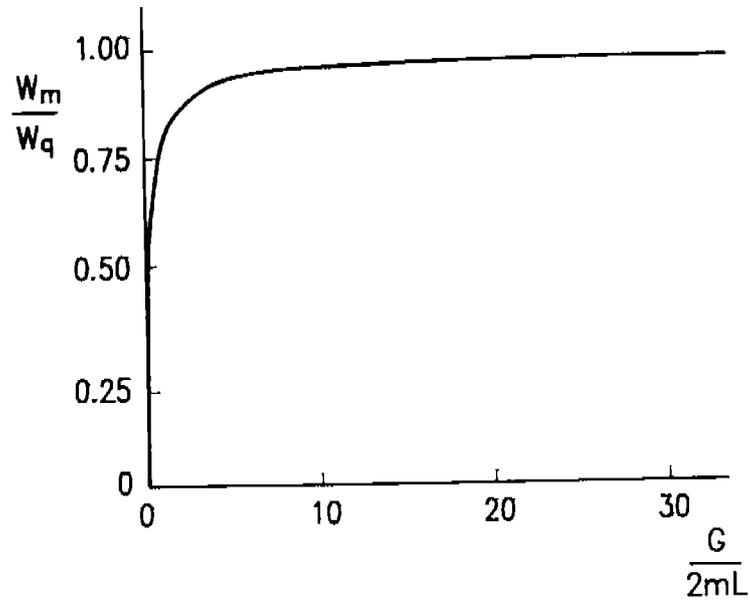


Figure 4

Ratio of the maximum permanent transverse displacements for the rigid, perfectly plastic beam in Figure 1 according to an exact dynamic analysis (W_m) (equation (8)) and a quasi-static analysis (W_q) (equation (10))

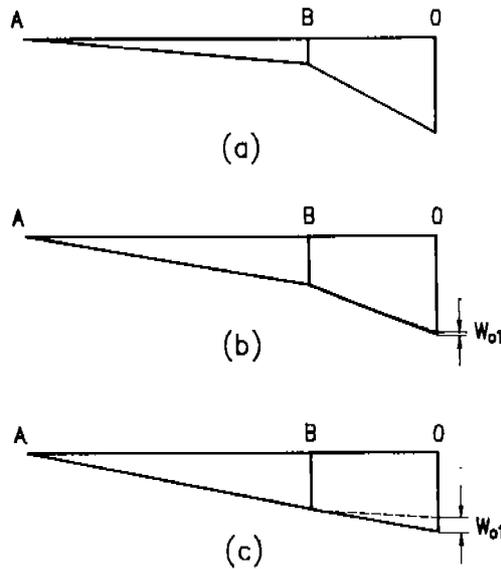


Figure 5

The final transverse displacement profiles of the main beam AB0 of the grillage AGi in Reference [23] for three initial velocities and a given external kinetic energy of 50 Nm according to the rigid-plastic analysis in Reference [24].

- (a) $V_0 = 5 \text{ m/s}$
- (b) $V_0 = 20 \text{ m/s}$
- (c) $V_0 = 50 \text{ m/s}$

W_{01} is the contribution to the maximum permanent transverse displacement from the travelling plastic hinges in the central half-span B0 of the grillage.

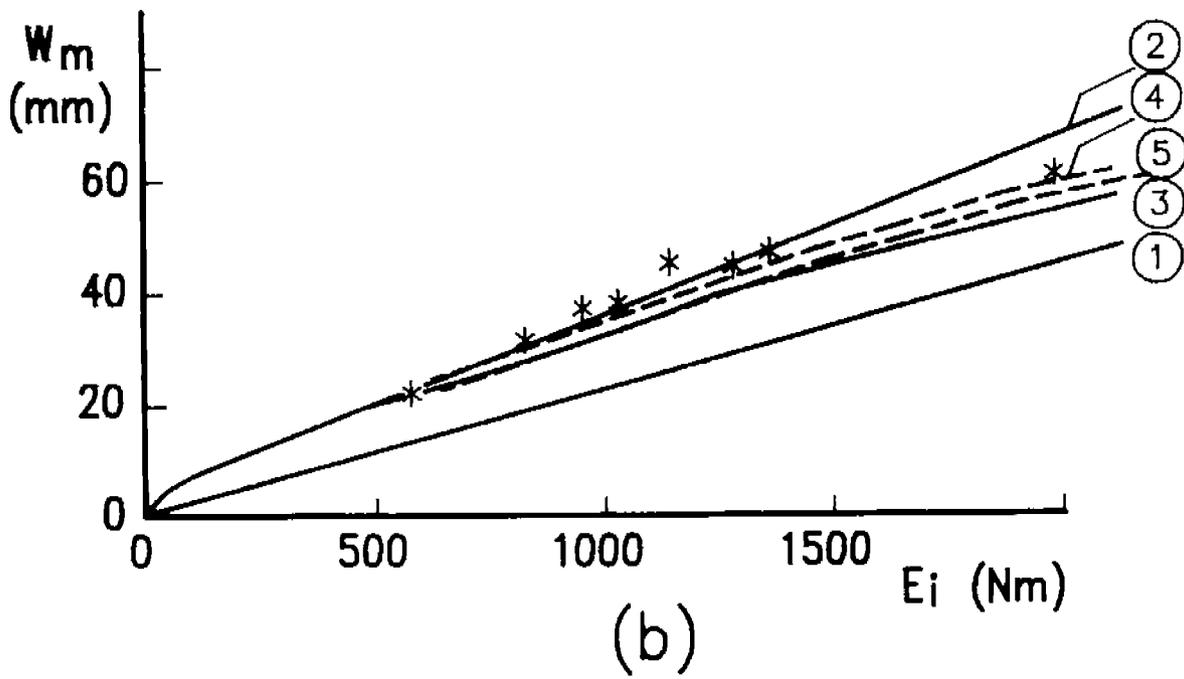
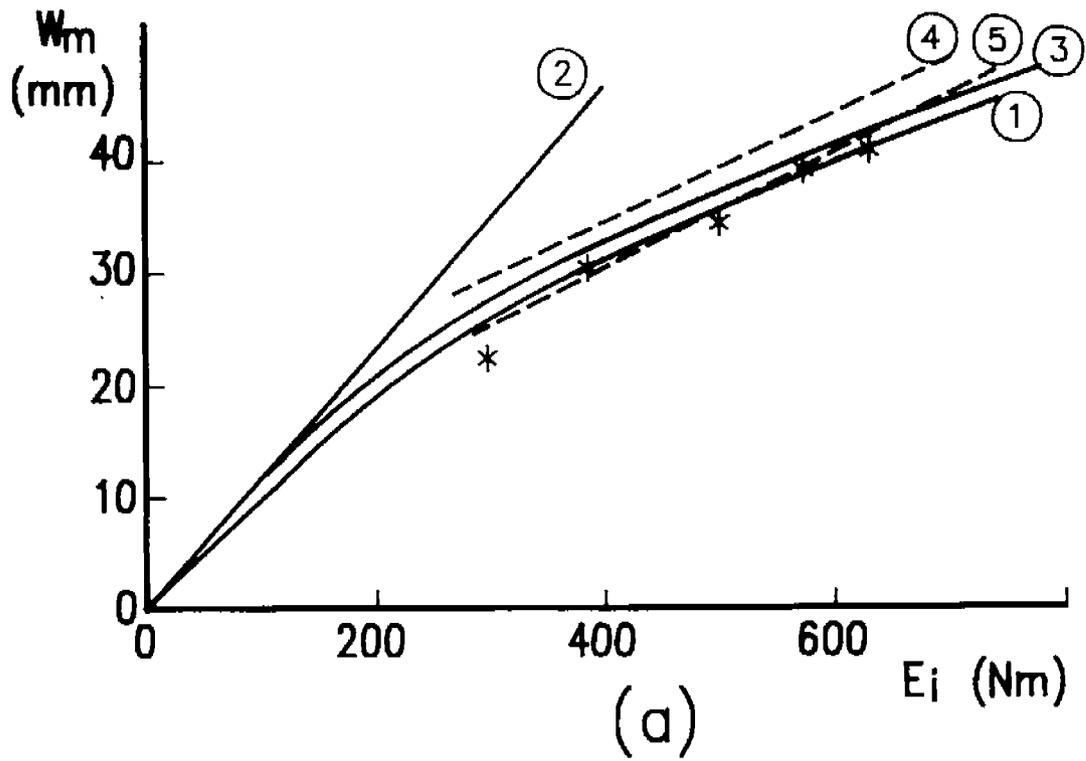


Figure 6

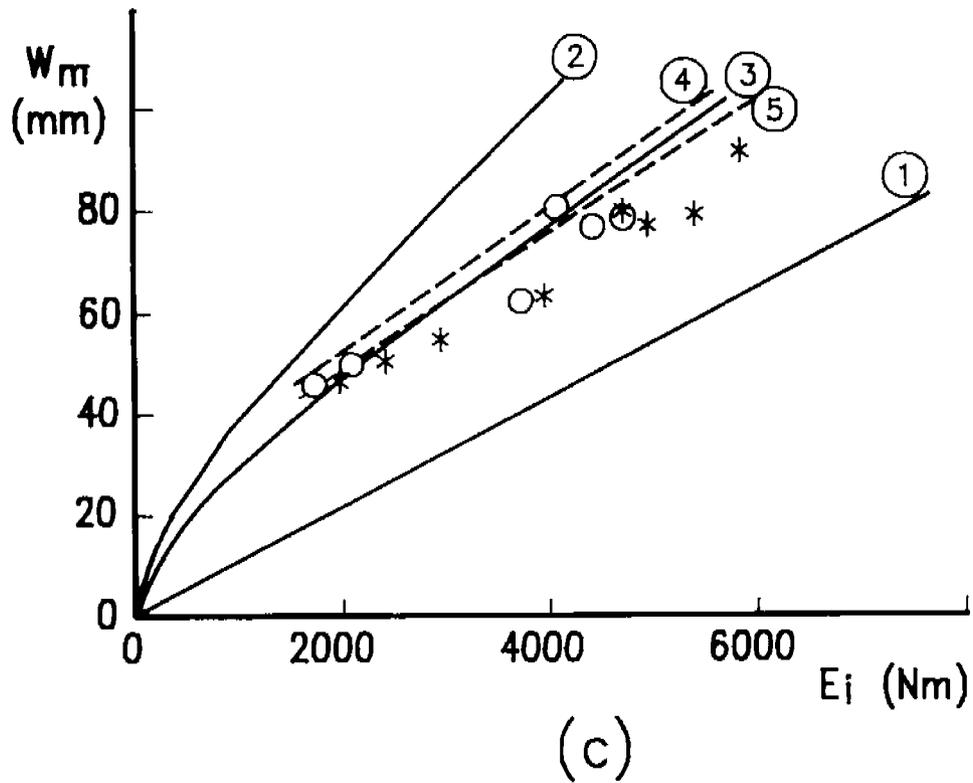


Figure 6 (continued)

Comparison of the theoretical predictions for the maximum permanent transverse displacements with the corresponding experimental results on fully clamped mild steel pipelines struck at the mid-span.

- * : experimental results [25]
 - O : experimental results with the rebound energy subtracted from the initial kinetic energy [25]
 - ① : theoretical predictions of Soreide and Amdahl [28]
 - ② : theoretical predictions of Ellinas and Walker [29]
 - ③ : theoretical predictions of Oliveira, Wierzbicki and Abramowicz [30]
 - - - - ④ : theoretical predictions of Reference [26] using yield stress σ_y
 - - - - ⑤ : as ④ but using the flow stress $(\sigma_y + \sigma_u)/2$.
- (a) $D/H = 11$
 (b) $D/H = 30$
 (c) $D/H = 60$

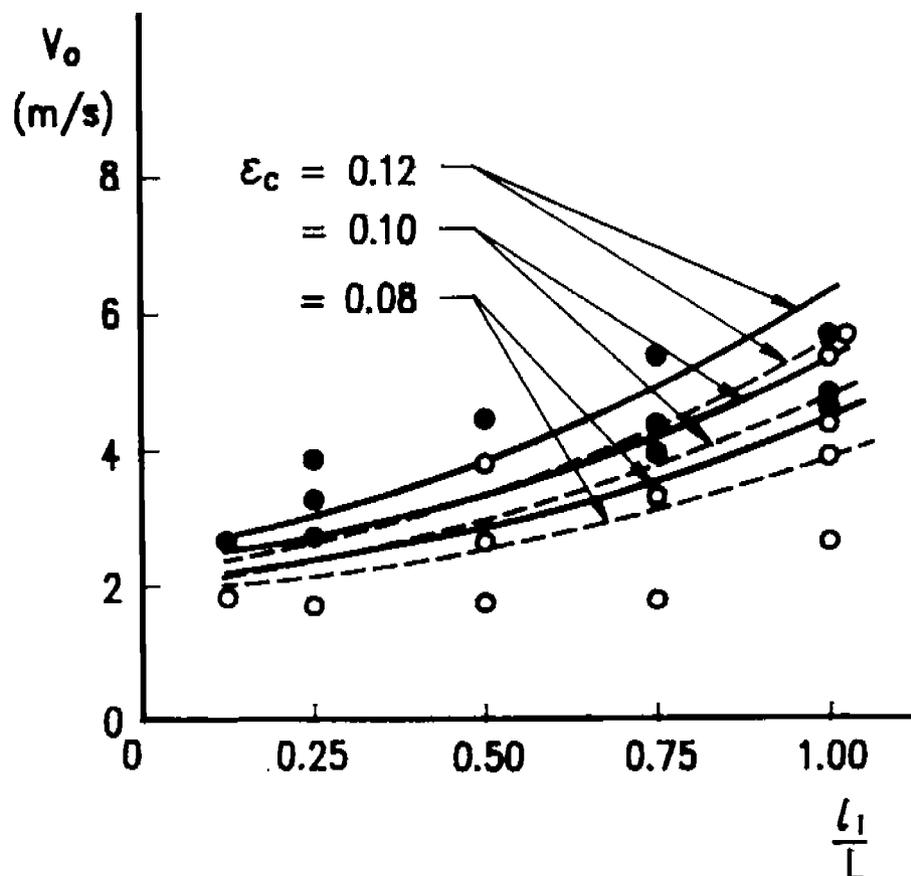


Figure 7

Comparison between the theoretical prediction [43] with several values of ϵ_c and the experimental results [13] for aluminum alloy beams with $H = 3.81$ mm and subjected to mass impact loadings at various locations on a fully clamped span

- : theoretical predictions [43] with $C = 6500$ s⁻¹ and $q = 4$
- - - : theoretical predictions [43] for a strain rate insensitive material
- : ruptured [13]
- : no failure [13]